JOURNAL OF AIRCRAFT
Vol. 41, No. 1, January–February 2004

Impact of Operations Research on the Evolution of the Airline Industry

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The airline industry can been described as one of the most technologically advanced. The primary players—the scheduled carriers rely on a wide variety of management information systems and decision support systems, which are typically developed using concepts from operations research and artificial intelligence. Operation research techniques have been applied to all areas of airline management, ranging from the strategic planning phase through to the post-analysis phase of operations. We present the state-of-the-art in the airline industry as it relates to applying operations research to solve traditional airline planning problems. We describe each business problem, review existing solution procedures, highlight recent research activity, and identify potential advancements and research opportunities. In addition, we quantify the benefits of implementing operations research based solutions in the airline industry.

Introduction

THE airline industry has been described by many as one of the most technologically advanced industries around today. The primary players, the scheduled carriers, rely on a wide variety of management information systems and decision support systems, which are typically developed using concepts from operations research and artificial intelligence. Operation research (OR) techniques have been applied to all areas of the airline industry, ranging from the strategic planning phase through to the postanalysis phase of operations. Like all other facets of today's society, the airline industry's reliance on computer technologies in conjunction with the application OR-based decision support systems has allowed carriers to increase productivity, reduce costs, and improve profitability. Airlines that are first to embrace new technologies often gain significant competitive advantages over the rest of the industry. Recent advances in mathematical programming techniques and increased computing power now enable airlines to solve problems that five years ago were thought to be unsolvable. There now exists the ability to solve large-scale decision models that encompass numerous disciplines within an airline, which have been traditionally dealt with independently. This in itself has transformed the very fundamental principles of the business.

Over the past 25-five years the global airline industry has been transformed from a highly regulated industry to one that is best described as a market-driven environment. This transformation has resulted in increased competition and the need for airlines to establish and sustain a competitive edge. International airlines are typically governed by bilateral agreements, which often restrict the type of aircraft that can be used on a specific route. The recent trend towards deregulated and liberalized air transport systems throughout the world has forced airlines to reassess the very nature of their existence with regards to their planning, analysis, and operations. Since deregulation in 1978, the U.S. domestic air transportation market has been transformed into a highly competitive environment marked by a rapid growth in the number of low-cost carriers entering the market. This fact has forced U.S. major network carriers to reengineer business practices in many functional areas, from the introduction of revenue management, the restructuring of their flight network, to the development of centralized airline operations control centers. This phenomenon has spread to other parts of the world and is now playing out in the intra-European market.

One of the major outcomes of the deregulation in the U.S. domestic market was the development of hub-and-spoke networks. Hub airports have developed at strategically located cities and are used as transfer points for passengers traveling from one community to another in the region surrounding the hub airport. They also serve as collection nodes for passengers traveling to and from the nearby communities to international gateways and other parts of the country. As a result, airlines can now provide service to more origin–destination markets. Typically carriers schedule a series of banks of flights into and out of their hub airports over the course of the day. Each bank normally consists of 20 plus aircraft arriving within minutes of each other, followed by a short period of no aircraft movement, and then the departure of all aircraft at the hub to their final destination. While the aircraft are on the ground, connecting passengers arriving on in-bound flights can transfer to their outbound flights within the prescribed connecting time. From an airline planning perspective the hub-and-spoke network has given carriers the opportunity to better manage the use of their limited resources, in particular aircraft and crew members. At the same time it has substantially increased the complexity of aircraft routing and crew scheduling, as the number of possible feasible solutions will increase because of the combinatorial nature of a highly connected network.

The recent introduction of next-generation regional jet aircraft has resulted in the rapid growth of many regional carriers. U.S. major network airlines such as Continental Airlines and Delta Airlines now rely heavily on their regional partners that fly fleets consisting of more than 50% jet aircraft. This has enabled these carriers to restructure their flight network to better meet market demand. In turn, this has had an impact on both crew scheduling and aircraft routing not only at the major network carrier, but also at the regional carrier. The regional carriers of today are now faced with longer stage lengths and increased aircraft utilization that have forced them to start using computer-based decision support systems to aid with the scheduling process. Network carriers now have to expand their strategic and tactical decision-making process to incorporate the needs and initiatives of their regional carriers. At the tactical level procedures such as yield management decisions have to be coordinated with the regional carriers as most network carriers have developed origin–destination systems that require input on all market served.

Received 24 February 2003; revision received 15 July 2003; accepted for publication 16 July 2003.

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Once considered independent powerhouses, many airlines now see the need to develop partnerships with domestic and international carriers in an effort to meet the market demand for seamless travel. The structure of collaboration ranges from franchising to global airline alliances such as the one world and the Star Alliance. The development of these global alliances will inevitably force airlines to change the way they schedule flights, assign crew members, and route aircraft. One notable impact will be the increased dimension of the decision model that will have to be solved if the partner airlines are truly committed to working toward a common goal. Significant cost savings exist, if airlines are willing and/or able to pool common resources that would allow each partner carrier to have access to more resources such as gates and ground systems equipment. In the interim individual carriers will have to rework many traditional approaches to schedule planning, and execution.

Airline planning has been impacted and will be further transformed by the development of aircraft families that share common cockpit crew rating, but have a wide range of seating capacities to better meet the needs of the market. In today's environment an airline is able to operate a fleet of Boeing 737s with capacities ranging from 103 to 182 seats. Similarly, another carrier could operate the Airbus 320 family, from the A318 to the A321, again having the wide range of seating options. The introduction of aircraft families has been driven by the desire of airlines to reduce direct operating costs, which is achieved from reduced crew-training costs, increased productivity, and reduced maintenance and inventory costs, as aircraft families generally share common engine types. However, the introduction of crew-compatible aircraft within an airline's fleet will increase the overall dimension of the crew-scheduling problem and could potentially allow the airline to offer better service through improved flight schedules.

In the subsequent paragraphs we present the state of the art in the airline industry as it relates to application operations research to solve traditional airline planning problems. We will describe each business problem, review existing solution procedures, highlight recent research activity, and identify potential future advancements. In particular, we will consider fleet assignment, yield management, crew scheduling, aircraft maintenance routing, operations control, and air-traffic-control slot management. In addition, we introduce several new concepts that have been recently developed including dynamic scheduling, hybrid airline scheduling, and integrated airline scheduling.

Traditional Applications of Operations Research in Airline Planning

The airline planning process has evolved into a distinct sequence of decision-making phases, where the output of a given phase is used as input to subsequent decisions. The scheduling process starts with schedule construction and fleet planning that is typically succeeded by four concurrent procedures, namely, crew scheduling, aircraft maintenance routing, airport resource management, and revenue management. Traditionally, these four decision-making initiatives are dealt with independently, with little if any interaction between the corresponding functional groups with the airline. The results of the aircraft maintenance routing process are then used during the tactical planning stage, where the airline has to make decisions about schedule recovery and manage its limited resources. An overview of the airline planning process is shown in Fig. 1, showing the chronological sequence of events that occur during the course of the decision making. The airline business processes and procedures highlighted in bold are discussion at length in this section.

There are two important criteria that affect how a flight schedule is built and developed: profitability and feasibility. The profitability of a flight schedule depends on its ability to attract revenue from passengers and cargo as well as on the inherent expense in operating it. Analysis of potential revenue comes from understanding the competitiveness of an airline's schedule in each origin–destination or pair of cities in which passengers or cargo can travel. In general, an airline will attract significant revenue if it offers relatively attractive service in origin–destination markets in which there are large flows of passengers or cargo. Major components of an airline's costs include crew, fuel, aircraft ownership, facilities, and other expenses. The feasibility of a schedule is based on the airline's ability to cover all flights with its pool of resources—aircraft, crews, and airport facilities. Flight schedulers typically begin the process of developing a flight schedule over one year in advance of publication by following a sequence of steps. These include identifying a basic schedule structure, or list of routes and frequencies; developing an initial feasible schedule as consistent as possible with that structure; and reviewing that schedule with various internal and external agencies and modifying it as appropriate. Throughout this process flight schedulers analyze possible changes to this schedule, such as new routes, new aircraft, different connecting opportunities, new frequencies, or a different hub orientation. Flight schedulers typically complete this process and publish the flight schedule, or submit it to computer reservation systems within 2–6 months before the schedule is first operated.

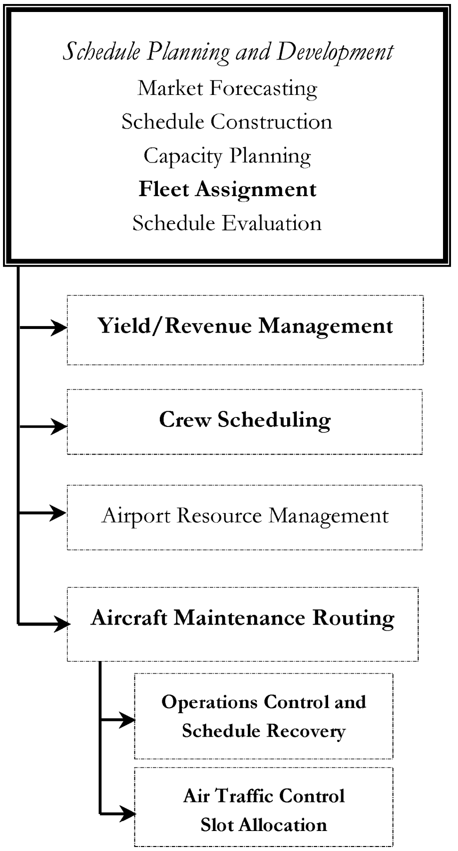

Fig. 1 Overview of the airline planning process.

Fleet Assignment

Given a predetermined flight schedule, the fleet assignment problem determines which aircraft type is assigned to a given flight segment in the carrier's network. The goal of the fleet assignment problem is to assign as many candidate flight segments as possible in a schedule to specific aircraft types, based on such factors as operating costs, revenues, and operational constraints and capabilities. The problem is formulated and solved as an integer-programming (IP) model that permits the assignment of multiple fleet types to a flight schedule simultaneously. The fleet assignment model can be classified as a large multicommodity flow problem with side constraints defined on a time-space network. One of the earliest published articles on the topic of fleet assignment was presented by American Airlines in Interfaces in 1989. Abara[1] discusses the application of integer linear programming to the fleet assignment problem and explains how this technique is used extensively throughout the carrier. Abara reports that an airline using a computer-based fleet assignment system using OR techniques can reduce operating costs by

0.5% and increase revenue by 1%. For a major U.S. network carrier these margins can translate to savings of over \$100,000 per day.

The fleet assignment model can be formulated as follows:

Minimize

$$\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \mid \text{flight} \qquad \text{(FAM1)}$$

$$\sum_{i \in I^+(n)} x_{ij} - \sum_{i \in I^-(n)} x_{ij} = 0 \qquad \forall n \in N, \quad j \in J \quad \text{(FAM2)}$$

$$\sum_{i \in I^- \mid \text{overnight}} x_{ij} \le \text{Number } Ac_j \qquad \forall j \in J \quad \text{(FAM3)}$$

$$x_{ij} \ge 0 \qquad \forall\, i \in I, \qquad j \in J$$

$$x_{ij} \in \{0, 1\} \qquad \forall\, i \in I \mid \text{flight}, \qquad j \in J$$

$$x_{ij} \in \text{integer} \qquad \forall\, i \in I \mid \text{overnight}, \qquad j \in J$$

Equality FAM1 represents the flight covering constraint that ensures that each scheduled flight is assigned to only one fleet type j. Equality FAM2 is a node balance constraint for equipment type j and each node n in the network. Inequality FAM3 ensures that the number of aircraft of type j used in the solution does not exceed the fleet size. Additional inequality constraints can exist on the decision variable x_{ij}, which addresses through-flight assignment, maintenance and crew considerations, slot allocation, and operational conditions such as noise restrictions.

Subramanian et al.[2] presents a solution procedure referred to as Coldstart, which is a fleet assignment methodology developed by Delta Airlines. Coldstart minimizes a combination of operating and passenger spill costs, subject to operational constraints. The solution strategy used in Coldstart employs an interior point method to solve the problem initially as a linear program. It then modifies the structure of the original problem by fixing certain variables and solves the resulting problem as a mixed integer program. Hane et al.[3] outlines a model for the fleet assignment problem and discusses some of the solution issues such as degeneracy, which often exist and lead to poor performance of standard Linear Programming (LP) solution techniques. The solution methodology presented incorporates an interior point algorithm, cost perturbation, model aggregation, branching on set-partitioning constraints, and prioritizing the order of branching, in an effort to develop more efficient solution procedures for the problem. A comparison is given on the performance of the solution procedure to standard LP-based branch and bound methodology. Clarke[4] discusses maintenance and crew considerations in the basic daily fleet assignment problem of Hane (1993) and implementations issues related to its solvability. The model generalizes the daily fleet assignment model to capture certain aspects of maintenance and crew scheduling. The solution methodology presented involves the use of the dual steepest edge simplex method, in combination with a customized branch and bound strategy.

The most recent advances in the fleet assignment problem have included the development of the origin–destination-based model that incorporates passenger flow issues more accurately into the decision model. The underlying assignment model is similar to the segment-based decision model, as it considers the same operating constraints. However, the enhanced decision model requires more specific forecasting data for each itinerary in the schedule network and is capable of differentiating between various multiple fare classes. The origin–destination fleet assignment model (ODFAM) better captures network dependencies that will allow airlines to manage their aircraft more effectively and efficiently. It is able to control passenger mix and reflect the upstream and downstream effects of the fleet assignment decision. It is anticipated that an airline using ODFAM will be in a better position to maximize revenues from connecting traffic, both online and from their code-sharing partner carriers.

Jacobs et al.[5] and Kniker[6] independently discuss the origin–destination fleet assignment problem that is now considered the state of the art in airline scheduling. Current research in the fleet planning arena is focused on the concepts of schedule generation and schedule recovery. Schedule generation encompasses the existing business processes of schedule construction and fleet assignment that have been traditionally solved sequentially. The concept of schedule recovery deals with real-time schedule adjustments in order to recover from disruptions. We will later discuss both concepts separately in this paper.

Yield (Revenue) Management

The yield-management process maximizes revenue by allowing an airline to selectively accept and reject reservation requests based on their relative value. The procedure is designed to effectively manage perishable seat inventory because an empty seat at flight departure cannot be resold. The control of reservation inventory to maximize revenues is normally accomplished through a series of sequential processes including overbooking, fare mix or discount allocation control, group control, and traffic flow control. Overbooking is the process by which additional reservations beyond physical capacity (seats) are sold to compensate for the effects of cancellations, no-shows, duplicate bookings, and misconnects. The practice of overbooking generally improves the passenger count onboard a flight and reduces the incidence of empty seats.

The primary objective of discount allocation control is to limit the number of seats sold to lower valued passengers by protecting seats for late-booking higher-valued passengers. This is achieved based on the results on an optimization-based decision support model. The optimal discount allocation controls result in the optimal onboard mix of full fare, discount, leisure, and deep-discount passengers to maximize the total onboard revenue. The process of yield management effectively manages the risk associated with this uncertainty and maximizes expected revenues. The technique used to calculate the authorization levels for the various subclasses is accomplished by calculating the marginal seat revenue. These computations do not consider up-line and downline displacement of traffic in the network, yet do reasonably well when the connecting traffic is minimal in the network.

Effective revenue management can result in 3 to 7% increases in operating revenues through the better utilization of existing assets. For a major U.S. network carrier this can translate to an additional \$500 million in annual revenues resulting from reduced spoilage (empty seats) and increased average yields. First-generation yield-management techniques were developed to maximize revenues on a leg-based inventory control scheme. Second-generation systems took a segment-based approach to the problem—the control of different fare classes and on-flight itineraries on a multiple leg flight. The current state of the art utilizes an origin–destination approach—process of allocating seats and determining booking limits for each specific origin–destination and fare class within a network of flights. In a full origin–destination inventory control environment reservation inventory is controlled by the actual origin and destination based on reservation value. This is accomplished using a network optimization model that takes the flight schedule, network capacity, and the origin–destination demand forecasts and variance by class to determine the probabilistic bid prices by leg and base compartment.[7] The bid price can be interpreted as the minimum acceptable fare for a reservation request on a flight leg to be accepted. The bid price for a multiple leg itinerary is the summation of the bid prices on each flight leg. Fares greater than the minimum acceptable fare or the total bid price are open for sale, subject to satisfying the associated fare rules. The fundamental difference between nested controls and origin–destination control is that availability is not prestored, but is dynamically calculated for each reservation request.

The revenue mix or discount allocation control optimization model can be formulated as follows:

Deterministic formulation:

Maximize

$$\sum_i P_i x_i$$

Subject to

$$\sum_i A_i^j x_i \leq \text{Capacity}_j \qquad \forall\, j \in J \qquad \text{(YM1)}$$

$$x_i \leq \text{Demand}_i \qquad \forall\, i \qquad \text{(YM2)}$$

$$x_i \geq 0 \qquad \forall\, i \qquad \text{(YM3)}$$

where $A_{ij} = \begin{cases} 1 & \text{if pax type } i \text{ uses flight leg } j \\ 0 & \text{otherwise} \end{cases}$

Inequality (YM1) represents the passenger mix constraints and J represents the list of all flights; the shadow price (dual variables on the dual problem of X) on these constraints becomes the bid price of the flights. Inequalities (YM2) and (YM3) represent the demand and nonnegativity constraints. Alternatively, the problem can be formulated in a probabilistic manner.

Stochastic formulation:

Maximize

$$\sum_i P_i \left[\int_0^{x_i} s f_i(s)\,\mathrm{d}s + x_i \int_{x_i}^{\infty} f_i(s)\,\mathrm{d}s \right]$$

Subject to

$$\sum_i A_i^j x_i \leq \text{Capacity}_j \qquad \forall\, j \in J$$

$$x_i \geq 0 \qquad \forall\, i$$

where $f_i(s)$ and P_i are respectively the demand density function and fare for passenger type i (class i).

Littlewood[8] was the first to present the marginal seat revenue method for determining discount seat allocation. Littlewood's approach involved forecasting future expected demand for each fare class by flight leg through applying statistical models to historical booking data on previous corresponding flight departures. Belobaba[9] discusses the development and application of a heuristic-based probabilistic decision model to airline seat inventory control. The concept referred to as expected marginal seat revenue (EMSR) takes into account the uncertainty associated with estimates of future passenger demand as well as the nested structure of booking limits in airline reservation systems. The EMSR model is used to set and revise booking limits periodically prior to flight departure. EMSR was the first widely applied optimization-based methodology for controlling the seat inventory. Group control is done using a group evaluator that assists in deciding whether to accept or reject the group booking. The group evaluator determines the minimum acceptable fare based on the expected displacement cost of individual passengers, projected group attrition forecast, the size of the group, the peripheral profit, and the number of complementary seats requested by the group.[10]

The process of traffic flow control is very important in an airline network with high levels of connecting traffic. The control of reservation inventory by origin–destination (itinerary) is accomplished using the value of the individual passenger to determine reservation availability. The passenger value is based on several factors including his itinerary, departure date, fare class, actual paid fare, and point of sale. The concept of virtual nesting was developed to approximately control reservation requests by origin–destination. It relies on the aggregation of various origin–destination fare classes that flow over a flight leg into a manageable number of virtual buckets based on reservation value. The value of an origin–destination class is the fare net of up-line and downline displacement costs. The buckets are serially nested to ensure that as sales build up for a flight; the lower-valued classes are automatically closed.[11] Curry[12] discusses an optimal airline seat allocation method that handles fare classes nested by origin and destinations. The solution procedure combines concepts from marginal seat revenue methods and mathematical programming to develop equations that find optimal allocation of seats when fare classes are nested on an origin–destination itinerary, and the inventory is not shared among origin–destination. The marginal seat revenue approach accounts for the nesting of fare classes in computer reservation systems, whereas the mathematical programming approach accounts for multiple origin–destination itineraries. The author shows the results for optimal booking limits for leg-based seat allocation with nested fare classes.

Williamson[13] provides a comprehensive review of the application of mathematical programming and network flow models to the origin–destination seat inventory control problem. Belobaba[14] reviews the evolution of airline yield management from fare class to origin–destination seat inventory control. The author highlights the major milestones in the airline yield management arena, discusses the origin–destination seat inventory control problem, and outlines a new solution approach that uses the minimum acceptable bid prices derived from leg-based optimization models to control seat availability. Talluri and van Ryzin[15] use a general model of the demand process to show that bid-price control is not optimal in general and analyze why bid-price schemes can fail to produce correct accept/deny decisions. They show that when leg capacities and sales volumes are large, bid-price controls are asymptotically optimal, provided the right bid prices are used. In a subsequent paper Talluri and van Ryzin[16] develop a randomized version of the deterministic linear programming method for computing network bid prices. The method consists of simulating a sequence of realizations of itinerary demand and solving deterministic linear programs to allocate capacity to itineraries for each realization. The dual prices from this sequence are then averaged to form a bid-price approximation. The authors show that this methodology can be used to approximate the optimal value function of the expected revenue.

Crew Scheduling

The topic of crew scheduling can be subdivided into several categories including crew-pairing generation, crew rostering, preferential bidding, and crew recovery. The crew-pairing problem consists of constructing a set of pairings that cover at minimum cost a given set of flight segments. Typically all flight legs pertain to the same aircraft fleet, and individual crew members are not considered in the problem. Each pairing has to be constructed in accordance with the prevailing collective agreement and airline regulations. The crew-rostering problem entails the construction of monthly work schedules that assign pairings and rest periods to each employee, while considering preassigned activities such as recurrent training, and personal holidays. The preferential bidding problem is a slight variation of the crew-rostering problem wherein employee preferences are incorporated into the crew-scheduling process. The crew-recovery problem consists of rebuilding broken crew pairings using existing pairing and reserve crews. After fuel costs crew costs are the second highest component of direct operating costs for an airline. The introduction of operation-research-based decision support systems has allowed airlines to reduce the percentage of pay and credit (an airline industry standard measure for crew utilization that measures the difference between actual and planned flying hours) by up to 50%. A 1% increase in crew utilization at a major U.S. network carrier can translate to over \$10 million savings per year.

The crew-pairing optimization model can be formulated as follows:

Minimize

$$\sum_{j \in J} c_j x_j$$

$$\sum_{j \in J} a_{ij} x_j = 1 \qquad \forall\, i \in I \qquad \text{(CPM1)}$$

$$x_j \in \{0, 1\}$$

where c_j is equal the overall cost of a pairing j and

$$a_{ij} = \begin{cases} 1 & \text{if flight } i \text{ is covered by pairing } j \\ 0 & \text{otherwise} \end{cases}$$

Equality CPM1 ensures that each flight leg is covered once and only once.

Gershkoff[17] outlines an optimization model that uses a set-partitioning framework, wherein the rows represent flights to be covered and the columns represent candidate crew trips. The primary objective of the model is to minimize the cost of flying the published airline schedule, subject to operational crew constraints. The crew-scheduling problem is formulated as an integer programming problem and can be solved using a commercial optimization software package. In these early efforts on the topic, it was found that a global optimization to the problem was difficult to achieve, and subsequent research focused on the development of efficient heuristic procedures to address the problem. Concepts such as dynamic column generation and LP relaxation play a major role in the ability of researchers to tackle and successfully solve such large-scale mathematical programs of crew-pairing optimization. Crew-pairing optimization has evolved substantially in the last 10 years, as advances in computer technology, CPU's time, and the availability of efficient optimization software packages have given practitioners the ability to solve larger problems than ever before.

The development of TRIP (trip reevaluation and improvement program) at American reflected an advancement in the solution methodology used in the area of crew scheduling and planning. Anbil et al.[18] discusses some of the operational benefits of implementing such a decision support system and its overall impact on the finances of the company. TRIP is based on a solution approach in which the crew pairings are iteratively improved by generating and solving a series of subproblems, whose performance will depend on the level of detail applied to the subproblems. In addition to the nominal role of developing crew pairing for the carrier, TRIP has been used by American as a general decision support tool to address problems related to crew staffing, such as the effects of crew-base closures on operations, manpower requirements at a given crew base, the economic impact of contract negotiations, and changes in operational rules. One of the driving characteristics of the crew-scheduling problem is the size of the constraint matrix in the mathematical formulation of the problem. Substantial research efforts have been spent studying the application of revolutionary solution techniques to aid in the timely solution of such large-scale problems. One of the most successful solution procedure that has been developed is the so-called branch-and-price technique, which combines the standard IP solution technique of branch and bound, and column generation. Another hybrid technique is the branch-and-cut procedure, which combines branch and bound, with cutting plane generation. In recent years, a number of articles have been published on these methodologies, and they are considered the leading-edge technology in the field of crew scheduling.

Barnhart and Shenoi[19] discuss a column generation technique for the long-haul crew assignment problem, which is solved by a two-phase process. A linear programming (LP) relaxation is applied to the problem, and the resulting linear program is solved using column generation.

Columns are efficiently determined using a specialized shortest path procedure. In the second phase of the solution procedure, an integer crew assignment is found using IP solution techniques such as branch and bound. The hybrid solution procedure of branch and price is a slight variation on the preceding methodology in that, during the course of the solution process, there are iterations between the two phases (steps) just outlined. At each node of the branch and bound tree, an LP relaxation is solved using column generation. Information obtained from the optimization portion of the method is used to modify the underlying problem, and the resulting problem is repeatedly optimized until a prescribed decision criterion is achieved. Hoffman and Padberg[20] outline a branch-and-cut methodology for crew scheduling. The branch-and-cut solver generates cutting planes based on the underlying structure of the polytope defined by the convex hull of the feasible integer points of the problem. The solver incorporates these cuts into a tree-search algorithm that uses automatic reformulation procedures, heuristics, and LP technology to assist in the solution process. The authors present case studies of both pure set partitioning problem and set partitioning problems with side constraints. Vance et al.[21] describe a methodology for finding near-optimal solutions to the airline crew-pairing problem. The solution approach uses a dynamic column generation scheme to identify crew work schedules combined with a customized branch and bound procedure that allows column generation to be performed at each node of the search tree.

Desaulniers et al.[22] discusses a common solution approach for the different crew-scheduling problems (crew-pairing generation, crew rostering, preferential bidding, and crew recovery) that arise at the planning and operations stages. They outline a formulation based on the set partitioning problem, where each column represents either a crew pairing or a crew member schedule. An optimization approach based on column generation is proposed, in which the subproblem is shown to be a constrained shortest path problem on an associated graph. The authors discuss the particular difficulties of each type of problem and ways of overcoming these difficulties. Stojkovic[23] describes the operational airline crew-scheduling problem that consists of modifying personalized planned monthly assignments of crew members during day-to-day operations. The model is used to generate modified pairing for selected crew members by simultaneously solving the classical crew-pairing problem and the problem of constructing personalized monthly assignments. The problem is formulated as a set partitioning problem and a column generation method embedded with in a branch-and-bound search tree is used during the solution procedure.

Although a large amount of research has been done on crew scheduling, the problem of crew recovery in the aftermath of irregular airline operations has only recent surfaced in the research community. It is important to point out that the majority of the existing crew scheduling models assume deterministic operating conditions. Current research in crew scheduling is focused on robust planning and crew recovery. Robust crew planning attempts to develop crew pairings that minimize actual pay and credit and are less prone to schedule disruptions.

Aircraft Maintenance Routing

The aircraft maintenance routing problem is traditionally solved after the successful completion of the fleet assignment problem. It involves the allocation of candidate flight segments to a specific aircraft tail number within a given subfleet of the airline. The process of aircraft maintenance routing has traditionally been a manual activity at many carriers, but in recent years researchers have developed efficient solution procedures that can be applied to the problem. During the normal operations of a carrier, situations often develop wherein modifications have to be made to the existing schedule plan. In addition, because of the inherent variation in passenger demand over the course of the week airlines find it necessary to adjust their daily flight schedules to adequately meet demand. This will result in the need to make minor modification to aircraft maintenance routings and possibly fleet assignments.

The aircraft maintenance routing model can be formulated as follows:

Minimize

$$\sum_{r \in R} c_r x_r$$

$$\sum_{r \in R} a_{\mathrm{lr}} x_r = 1 \qquad \forall\, l \in L \qquad \text{(MRM1)}$$

$$\sum_{r \in R(a)} x_r \le 1 \qquad \forall\, a \in A \qquad \text{(MRM2)}$$

$$x_r \in \{0, 1\}$$

where c_r is equal the total operating cost of routing r and

$$a_{\mathrm{lr}} = \begin{cases} 1 & \text{if flight leg } l \text{ is covered by routing } r \\ 0 & \text{otherwise} \end{cases}$$

Equality MRM1 ensures that each scheduled flight is covered. Inequality MRM2 ensures that each aircraft a is assigned to only one feasible routing.

Bard et al.[24] explore aircraft maintenance routing while taking into consideration the benefits of through-flight schedules and the potential for increased revenues. The authors develop an algorithm that can be used to efficiently pair inbound and outbound routes at hub cities over the course of the day. The methodology is based on a forward-searching heuristic designed to transcend the combinatorial difficulties that arise in aircraft routing problems for major domestic carriers operating a hub-and-spoke network. Talluri[25] describes an algorithm for making aircraft swaps that will not affect the equipment type composition overnighting at various stations throughout the airline's network. The algorithm repeatedly calls a shortest-path algorithm, and the performance of the swapping algorithm is a reflection of the availability of very fast shortest-path algorithms. He also outlines the application of the swapping procedure in the airline schedule development process.

Soumis et al.[26] presents a model for large-scale aircraft routing and scheduling problems that incorporates passenger flow issues. The solution methodology proposed is a heuristic adaptation of the Frank–Wolfe algorithm for an integer problem with a special structure. The procedure involves solving alternatively the aircraft routing problem and the passenger assignment problem until a prescribed criterion is satisfied. The authors discuss the technique used to transfer information from the passenger flow problem to the aircraft routing problem. Daskin and Panayotopoulos[27] discusses a Lagrangian relaxation approach to an integer linear program model, which is used to assign aircraft to routes in a hub-and-spoke network. They outline the Lagrangian solution procedure, as well as heuristics for converting the Lagrangian solutions into primal solutions to the problem. The research results suggest that the Lagrangian relaxation approach is effective at providing an upper bound on the objective function, and the heuristics can yield good solutions when there are adequate aircraft available to meet the passenger demand. Zhu et al.[28] present a mathematical formulation for the aircraft rotation problem and discuss its similarity with the asymmetric traveling salesman problem. Their solution procedure employs Lagrangian relaxation and subgradient optimization.

Kabbani and Patty[29] discuss aircraft maintenance routing at American Airlines and the application of mathematical programming techniques to solve the problem. Their initial approach to the problem was to formulate it as a set partitioning model, where the columns were constructed to represent each possible week-long routing. This formulation did not take into consideration maintenance constraints, and the solution procedure had a poor performance. The authors further outline a modified solution procedure in which the decision process in divided into two separate subproblems: the first subproblem dealing with the solution of appropriate daily aircraft turns and the second with connections among these daily routings. The solution methodology also makes use of heuristic procedures, if the sequential approach is not successful in determining all aircraft rotations for the fleet. Desaulniers et al.[30] outline the daily aircraft maintenance routing and scheduling problem and present two different formulations of the problem. The first is a set partitioning-type formulation, and the second a time-constrained multicommodity network flow formulation. A Dantzig–Wolfe decomposition approach is used to solve the linear relaxation of the multicommodity flow problem. In addition, they discuss alternative branching strategies that are compatible with the column generation technique and show the compatibility between the two different formulations.

Operations Control and Schedule Recovery

Schedule recovery in the aftermath of irregularities addresses how airlines reassign operational aircraft to scheduled flights. The main aspect of recovery for an airline is centered around flight rescheduling. For a typical airline approximately 10% of its scheduled revenue flights are affected by irregularities, with a large percentage being caused by severe weather conditions and the associated loss of airport capacity. The financial impact of irregularities on the daily operations of a major U.S. network carrier can exceed $400 million per annum in lost revenue, crew overtime pay, and passenger hospitality costs.[31]) Today, flight rescheduling is typically done manually at airlines, as researchers have not been able to thoroughly solve the problem of schedule recovery. Recent advances in mathematical programming and computer processing speed now enable researchers to consider solving real-time decision problems such as schedule recovery.

Teodorovic and Stojkovic[32] discuss a greedy heuristic algorithm for solving a lexicographic optimization problem that considers aircraft scheduling and routing in a daily schedule while minimizing the total number of cancelled flights in the network. The algorithm developed is based on dynamic programming and is characterized by a sequential approach to solving the problem as flights are assigned to aircraft in sequences. Jarrah et al.[33] present an overview of a decision support framework for airline flight cancellations and delays. Two separate network flow models provide solutions in the form of a set of flights delays (the delay model) or a set of flight cancellations (the cancellation model), while allowing for aircraft swapping among flights and the utilization of spare aircraft. Both models are solved using Busacker–Gowen's dual algorithm for the minimum cost flow problem in which the shortest path is solved repeatedly to achieve the necessary flow in the network. The network models presented are solved independently of each other and do not take into consideration crew and aircraft maintenance constraints.

Yan and Yang[34] develop a decision support framework for handling schedule perturbations. The framework is based on a basic schedule perturbation model constructed as a time-space network from which several perturbed network models are established for scheduling following irregularities. The authors formulate both pure network flow problems that are solved using a network simplex algorithm and network flow problem with side constraints, which are solved using Lagrangian relaxation with subgradient methods. They outline the basic schedule perturbation model that is designed to minimize the schedule-perturbed period after an incident, while maximizing profitability. Cao and Kanafani[35,36] discuss a real-time decision support tool for the integration of airline flight cancellations and delays. They present a quadratic 0-1 programming model for the integrated decision problem, which maximizes operating profit while taking into consideration both delay costs and penalties for flight cancellations. Arguello et al.[37] present a time-band optimization model for reconstructing aircraft routings in response to groundings and delays experienced in daily operations. This model is constructed by transforming the aircraft routing problem into a time-based network in which the time horizon is discretized, resulting in an integral minimum cost network flow problem with side constraints. The problem is initially solved as a relaxed linear programming problem and if necessary a mixed integer problem.

Clarke[31] discusses the airline schedule recovery problem that provides a comprehensive framework for reassigning operational aircraft to scheduled flights in the aftermath of irregularities. It is formulated as a mixed-integer LP problem and solved using an optimization-based solution procedure. The mathematical formulation of the problem enables flight delays and cancellations to be considered simultaneously. The decision model allows for multiple-fleet-type aircraft swapping during rescheduling and incorporates the impact of air-traffic-control flow management initiatives and crew availability. The model is based on a time-space representation that allows the use of efficient tree-searching algorithms to quickly solve the underlying subproblem of finding the best possible aircraft routing, subject to operating constraints. Lettovský[38] outlines the airline integrated recovery problem that addresses crew assignment, aircraft routing, and passenger flow. The problem is formulated as a mixed-integer LP problem and solved using Bender's decomposition. The master problem generates a cancellation and delay plan that satisfies imposed landing restrictions and assigns equipment types. The decoupled aircraft and crew subproblems are solved, then the passenger flow subproblem is solved to determine new itineraries for disrupted passengers. The author discusses the problem of crew recovery and describes a mathematical programming based solution methodology that optimally reassigns crews to flights legs while minimizing the additional cost and operational difficulties to the airline. The solution strategy initially identifies a set of eligible crews, whose original assigned unflown flight segments are used to form new crew pairings that are then reassigned to individual crews through a set-covering problem.

Air-Traffic-Control Slot Allocation

The ability of an airline to recover from severe weather conditions will depend on its interaction with the air-traffic-control system. Under such conditions air traffic control (ATC) typically imposes restrictions on aircraft movement at affected airports and implements what is generally referred to as a slot allocation scheme, as well as ground-delay programs. The response of the airline to these imposed conditions will be based on available data in the system operations control center. The use of a decision support system for ATC slot swapping can significantly reduce delays associated with air traffic control. A major U.S. network carrier reported that for a given year their decision support system (DSS) system saved them approximately 345,000 delay minutes, translating into $5.2 million savings in direct operating costs.[39] The guidelines governing ATC slot substitutions have been recently changed to help accommodate the operating needs of carriers in the air-traffic-control system. Most of the earlier published literature on the topic of slot allocation has been rendered obsolete, as changes to the substitution guidelines have significantly altered recovery procedures in use at airline operations control centers.

Vasquez-Marquez[39] discusses a network-based optimization system that was implemented at American Airlines to help reduce delays imposed by air traffic control. The arrival slot allocation system uses a network-based heuristic procedure to reschedule arrivals into affected airports, as ATC has developed slot substitution rules that allow airlines to use the arrival slots of cancelled flights to reduce delays created by the ground delay programs. The underlying model is based on a mixed-integer mathematical formulation and is solved using network flow theory. The model is treated as a directed traveling salesman problem, and the solution methodology uses a tour building heuristic approach designed to preserve aircraft, crew, and gate connections among flights at hub airports. The procedures are capable of handling multiple cancellations for which the problem becomes a multiple directed traveling salesman problem.

United Airlines has developed the slot allocation model (SLAM) that uses an assignment problem formulation to represent the allocation of landing slots to candidate arriving aircraft. The primary objective of the model is to minimize delay costs in the airline's system, while satisfying Federal Aviation Administration's regulations. The solution procedure of SLAM is based on a minimum cost flow algorithm that achieves optimality for any number of flight cancellations. A similar assignment model has been developed by USAirways, where the solution procedure is based on a minimum cost flow algorithm.

Luo and Yu[40] present the airline schedule perturbation problem under the ground delay program with restrictions on the flow of resources in the network. The authors propose strategies that can be implemented to address a specific schedule perturbation such as the ground delay program. They outline solution procedures for solving the perturbation problems that are based on assignment algorithms and more efficient algorithms, which rely on the detailed structure of the problem. They consider the general case of the ground delay program and its implications to the airline schedule perturbation problem. The general problem is modeled as an integer programming formulation with no restrictions on the flow of resources, and it considers both aircraft landings and takeoffs in the solution process. The solution methodology involves an LP relaxation of the problem and the application of a heuristic procedure to find feasible integer solutions. It is based on a polynomial algorithm for minimizing delay among outbound flights in the network.

Cao and Kanafani[35] consider the value of runway time slots for airlines in the aftermath of irregularities. They analyze the relationship between rescheduling flights and airline profitability, using a minimum cost flow model. Based on the model solution of the revised schedule of flights, the authors assert that the value of a given time slot at a specific station can be assessed. The model is validated using a sample airline flight network, and the authors also outline possible uses of this methodology by airlines, such as congestion pricing and runway slot auctioning.

Carlson[41] discusses the impact of ATC slot allocation on the scheduling of flight banks at a hub airport. In particular, the author looks at the assignment of banks of flights to arrival slots in a partially decentralized air-traffic-flow management environment. He outlines the mathematical formulation of a series of three decision models, which incorporate aircraft bank dependencies induced by connecting passenger flow at hub airports. Bertsimas and Stock Patterson[42] discuss the air-traffic-flow management problem with en-route capacities. They present a decision model that takes into account all of the capacitated elements in the system (arrival, departure, and sector capacity) and extends to incorporate the dependence of airport runway capacity of departures and arrivals, hub connectivity, banking, and rerouting flights when capacity levels drop drastically. The model determines the ground delay and airborne delay for each scheduled flight in order to minimize the total delay cost in the system.

Recent Applications of Operations Research in Airline Planning

The airline planning process has traditionally been viewed as a well-defined sequence of business activities that rely on established information flows. This approach has been reinforced by the underlying structure of the functional divisions within an airline. The sequential approach to airline planning has resulted in each business group trying to achieve resource-specific objectives that often affects the outcome of downstream decisions. Additionally, there is very little feedback to upstream processes that could help improve the integrity of the final airline schedule. An airline can be viewed as a collection of working networks—aircraft, crew members, passengers, and freight cargo—all connected by scheduled flights. Events that occur in one network will impact another network. As such, it is becoming more and more apparent that it important to address each planning problem in light on its impact on other planning problems.

Dynamic Scheduling

Throughout the course of daily operations, airlines face a major operational problem in assigning aircraft capacity to flight schedules to meet fluctuating market demands. Berge and Hopperstand[43] discuss the demand-driven dispatch operating concept that attempts to address this problem. Utilizing up-to-date and more accurate demand forecast for each scheduled departure, aircraft are dynamically assigned to flights in order to better meet anticipated passenger demand. The solution procedure requires the frequent solution of large aircraft assignment problems, which are formulated as multicommodity network flow problems and solved with heuristic algorithms. The authors outline case studies of actual airline systems in which increases in passenger loads are achieved, along with reductions in operating costs, resulting in a net improvement in operating profit. From a conceptual standpoint the potential can exist to conduct aircraft swapping with multiple aircraft types (different crew rating).

Gershkoff[44] presents the adaptive aircraft assignment (A^3) framework as a new paradigm in airline scheduling. The author describes the scheduling approach as a means to combine the characteristics of a scheduled carrier with that of a supplemental carrier to produce a more flexible dynamic carrier for long-haul services. The A^3 concept consists of six phases, ranging from deciding which flights to cancel to dynamically substituting aircraft to meet fine-grain demand variations. The adaptive aircraft assignment procedure is hybrid combination of the fleet assignment model, aircraft routing model, and concepts from yield management. Jarrah et al.[45] discuss modifications to the traditional fleet assignment model formulation that enables an airline to consider refleeting decisions. Airline planners often find it necessary to modify fleet assignments to react to inevitable changes in the competitive landscape that relate to the planned schedule, demand forecasts, number of available aircraft, crew staffing levels, and other scheduling constraints. Such changes require quick solution time to allow frequent refleeting analysis and constraints to minimize the deviations from the current fleet assignment, which drives all aspects of the airline business.

Hybrid Scheduling Problems

In recent years there has been a trend towards addressing hybrid airline problems such as the combination of the aircraft assignment and routing problem and the combined fleet assignment and crew-scheduling problem. However, these hybrid problems have been considered only for the strategic phase of the airline planning process. Researchers have started to explore so-called hybrid strategic planning problems, combining different phases of the airline planning process, which have been traditionally considered in sequential order. One such problem is that of the combined aircraft fleeting and routing problem. Barnhart et al.[46] and Shenoi[47] discuss a model and solution approach to solve simultaneously the fleet assignment and aircraft routing problems. The solution methodology incorporates costs associated with aircraft connections and complicating constraints (such as maintenance requirements and aircraft utilization restrictions), which are usually ignored in traditional fleet assignment solution procedures. The model is string based, and a branch-and-price solution approach is used to solve the problem. As described by the authors, a string is a sequence of connected flights that begins and ends at a maintenance station, satisfying flow balance, and meets the required maintenance constraints.

Farkas[48] and Belobaba and Farkas[49] considers the impact of network effects and revenue management on airline fleet assignment decisions. The author discusses an iterative solution process based on concepts from the traditional revenue management process and fleet assignment problem. Desaulniers et al.[50] discuss a unified framework for deterministic time-constrained vehicle routing and crew-scheduling problems developed by researchers at the GERAD Institute. The hybrid scheduling problems are formulated as multicommodity problems and solved using an extension to the Dantzig–Wolfe decomposition technique. Desaulniers et al.[51] formulate an integer-programming-based model that simultaneously schedules vehicles and crews to accomplish a set of tasks at minimum cost. The model is solved using a column generation approach where only crew schedules are generated, and vehicle schedules are derived in a polynomial time postprocessing phase. Belanger et al.[52] discuss the airline fleet assignment problem with time windows that incorporates issues related to schedule construction and frequency generation into the fleet assignment problem. Barnhart et al.[53] present an incremental airline schedule design problem whose objective is to improve airline profitability by adding new flights, deleting less profitable flights, or rescheduling flights. The authors assume an a priori knowledge on demand variation as a result of to schedule changes and that their model formulation captures only the first-order interaction between supply and demand.

Integrated Airline Scheduling

We envision that future airline schedule planning will be done in a more integrated fashion in an effort to improve operational efficiencies. Schedule planning and development decisions are currently done sequentially, which means limited information is shared, and there is very little interaction between the corresponding divisions within the airline. Upstream decisions on fleet planning and aircraft assignment will have a significant impact on downstream business decisions such as aircraft maintenance routing, crew scheduling, and revenue management. Traditionally, the overall airline schedule planning process has been divided into discrete phases because of technical limitations in computer hardware and the very culture of the airline business. However, as airline executives see the benefits of integrated planning and operations, airlines will be forced to embrace integrated planning.

Phillips and Boyd[54] discuss a framework for integrated airline fleet and schedule planning that addresses asset acquisition, asset utilization, and product offering decisions. The procedure is a hybrid of several scheduling phases that are traditionally solved sequentially. These include fleet planning, aircraft assignment, and route planning. The solution methodology is based on a linear program that optimizes the allocation of a fixed fleet to a schedule. Marginal values calculated by the linear program guide the addition and deletion of aircraft and help determine schedule adjustments to improve profitability.

Lettovský et al.[55] and Smith[56] describe the schedule generation model as a revolutionary approach to airline scheduling. The decision model allows users to analyze and improve existing airline schedule or to generate a new schedule—by making recommendations regarding schedule changes such as additions of routes, frequency modifications, timing adjustments, and equipment group assignments. An equipment group is defined as a collection of equipment types that share common characteristics, such as turboprops, jets, narrow bodies, and wide bodies. In effect, the schedule generation model has been designed to automatically generate the structural elements of a more profitable and feasible base schedule that serves as the foundation of the airline schedule development process.

Traditionally, the base schedule is based on historical timetables that are adjusted to meet future passenger demand. The structural elements include frequency targets or proposed operating frequencies for each route in a schedule, an approximate timing for each frequency, as well as individual origin point of presence targets for each equipment group and station combination. The model employs a global perspective to derive frequency and point of presence targets, taking into account each route's revenue contribution to the profitability of the total route network. The model takes as input scheduling constraints, origin–destination market sizes and revenues, competitive schedule information, and a host airline's schedule. The schedule generation model can be used in conjunction with the fleet assignment model through an iterative process to develop flight schedules that better meet market demands.

Impact of Operations Research on Airline Planning

The application of operations-research-based decision support systems in the airline industry has been driven by the desire to reduce costs, increase operating revenue, and ultimately improve profitability. In heavily regulated environments airlines were able to survive inefficient operations that were artificially supported by government bureaucracy. Very little concern was given to operational efficiency, as carriers were easily able to recoup their expenses with higher regulated airfares. With the advent of effective competition—a direct outgrowth of deregulation, airlines around the world, and especially those operating in the U.S. domestic market have had to reinvent themselves. Manual procedures have been gradually replaced with computer-based solutions that leverage the power of operations research, as well as other technologies such as artificial intelligence. As a result, airlines have been able to reduce the number of aircraft and auxiliary resources (gates, ground staff, crew members) required to operate a proposed flight schedule. In addition, airlines that embrace decision support tools have been able to substantial reduce their overall planning cycle.

Schedule Planning

In the crew and fleet planning arenas airlines that utilize optimization-based decision support systems have experienced substantial reduction in operating costs, as they are better able to match their resources (aircraft and crew members) with market demand. The development and implementation of a crew management system are required for any marginally sized airline—with more than 20 aircraft, as the manual planning process would be very cumbersome. The level of sophistication of the crew management system will be directly correlated to the size of the carrier. When American Airlines first implemented an optimization-based crew pairing system in 1989, they estimated that the cost savings of this approach relative to the previous enumeration methods was on the order of USD $18 million per year.[18] More recently, Ryan et al.[57] has reported saving over NZ$ 15,655,000 per year in crew scheduling costs at Air New Zealand while providing rosters that better meet crew member preferences—based on the utilization of optimization-based decision support systems developed in conjunction with the University of Auckland.

Within the major airlines of the world, the percentage of carriers using an optimization–based fleet assignment process lies at approximately 80%, as some airlines find it unnecessary based on their fleet composition. If an airline has only one type of wide-body aircraft for long-haul operations and one type of narrow-body aircraft for short operations, there is no need for automated fleet assignment. On the other hand, if an airline has amassed a number of fleet types within its fleet an efficient fleet assignment procedure becomes very important in determining the carrier's profitability. According to Subramanian,[2] when the Delta Airlines first implemented Coldstart—fleet assignment module in the fall of 1992, the planning group estimated that they had gained a cost savings of more than USD $100,000 per day. The cost savings from the utilization of the fleet assignment model increased as planners gain more experience and confidence with the module, reaching an estimated USD $200,000 per day in late 1993. Along the way there was some resistance to accepting the module's recommendations, as it changed the traditional way that fleet planning was done at the airline.

Revenue Management

The concept of revenue management within the airline industry dates back to the 1970s, when carriers introduced discount seats that were managed primarily by overbooking procedures. During the early 1980s, the gradual liberalization of the industry—ranging from the proliferation of fares to increased schedule variation—caused existing revenue management systems to stop working effectively. The development of first-generation revenue mix control models was not able to keep up with the changing environment. These systems relied on relatively simple solution methodologies and were based on assumptions that were quickly being erased by the advent of deregulation. The wide variety of airfares offered in the marketplace, often inhibiting revenue management systems from functioning properly. As a result, airlines were often required to realign fares offerings, in order to remove inconsistencies in fare classes that resulted from fare overlap.

The restructuring of airline network from point-to-point to hub-and-spoke changed traffic mix from 90% nonstop passengers to 40%. This phenomenon introduced the origin–destination dimension to the revenue maximization problem, which required several stop-gap mechanisms to be developed along the way including virtual nesting. In the aftermath of airline deregulation, technological innovation in revenue management systems has slowed to the point that new research concepts and ideas are obsolete by the time they go live in production systems. The development and implementation of next generation systems often take as much as 10 years to complete and is sometimes limited by available computer technology. An important issue that affects the deployment of new systems is that of smooth transitioning, which airlines view as essential, regardless of the envisioned improvement of revenues from the new revenue management system.

From the onset the practice of revenue (yield) management has allowed airlines to gain significant competitive advantages over other carriers—especially ones not using a revenue management (RM) system. Today, most airlines will agree that having an effective RM system is not a luxury, but a necessity. In fact, the top 100 airlines of the world all use a revenue management system in some form or the other. Technological advances in revenue management system are a direct outgrowth of OR research—enabling the initial leg-based approach to be generalized to the network-based approach. In reality, this approach is more representation of actual airline operations. According to Robert Crandall [former Chief Executive Officer (CEO) and chairman of AMR]:

> The development of the American Airlines yield management system had been long and sometime difficult, but this investment has paid off. We estimate that YM generated USD $1.4 billion in incremental revenue between 1989 and 1992. This was not a one-time benefit. We expect yield management to generate at least USD $500 million annually.[11]

Since the inception of revenue management systems, their overall impact on operating revenue has been reduced by the continued interference of analysts who often manually override the booking limits determined by the system. In addition, the accuracy of revenue management systems is highly dependent on the quality of the underlying forecasting mechanism in the system. In recent years researchers have started to ponder the impact of the worldwide web on revenue management, as airlines attempt to gradually change their primary distribution channel. The growth of online ticketing and auctioning schemes could have a substantial impact on existing forecasting techniques and overbooking mechanisms that were designed based on historical market behavior. The rapid changes that are occurring in the airline industry will require carriers to continually upgrade their decision support systems to meet operating conditions.

Tactical Planning

Although progress has been made in the area of schedule recovery, airlines have been slow to embrace optimization-based decision support systems designed to assist them with real-time planning. Such resistance is based on a variety of factors, ranging from the complexity of the underlying problem to the required approach to solve the schedule recovery problem. Airlines have traditionally been divided into functional groups based a prescribed resource—such as aircraft, crew members, and passenger services. In the past decision support systems have been designed to handle one resource, often ignoring auxiliary factors during decision making. Unfortunately this has created an environment of mistrust within airline operations control centers towards all decision support systems. Researchers have proposed that the most effective way to handle schedule recovery is to employ an integrated approach that transcends existing airline organizational structure.

Over the last 15 years airlines have identified the need to develop more integrated enterprises that share information and decision making across the organization vs the traditional "silo" approach. It is becoming evident to airlines that this integrated approach is not only beneficial during the tactical planning phase but also during the strategic planning period. Airlines have started to explore dynamic scheduling concepts that leverage yield management information for near-term fleet assignment decisions. Continental Airlines uses this concept for their Boeing 737 fleet that consisted of the B737-300 (124 seats), B737-500 (104 seats), B737-700 (124 seats), B737-800 (155 seats), and the B737-900 (167 seats).

Beyond airline resource planning, operations-research-based decision support systems have been slow to develop in other major mission critical planning area such as pricing and schedule development. Past efforts to address this deficiency have been limited by insufficient data and the ability to predict competitor's reaction to host airline decisions. The lack of adequate data sources exists in part because of the slow speed of the airline revenue accounting process and the manner in which market demand data is collected and forecasted. In the case of pricing, the evolving need for decision support systems is driven by the complexity of the underlying problem and the inherent synergies that exist between scheduling, revenue management, and pricing. Such systems will be required for an airline to be better able to respond to market conditions and conduct comprehensive analytical market studies, such as fare structure evaluation, cross-market comparisons, and estimating demand characteristics. Operations-research-based pricing decision support systems could assist airline in introducing fare changes, as well as responding to competitive fare changes. It is believed that the development of efficient pricing decision support systems will be one the next major advancements within the airline industry. As discussed earlier, the introduction of decision support systems for schedule development will most likely have a significant impact on the future airline planning, as the base schedule forms the foundation from which all other airline planning activities are derived.

Conclusions

The application of operation-research (OR) techniques in the airline industry has been fostered the Airline Group of the International Federation of Operational Research Societies (AGIFORS). AGIFORS organizes an annual symposium as well as series of study group meetings that cover various aspects of the airline planning process—airline operations, cargo, crew management, revenue management, and schedule planning. In addition, the application operations research in the airline planning process has been the subject of many research projects around the world. There have been countless dissertation and thesis topics related to the various airline business processes outlined in this paper. Besides providing a financial benefit to airlines, some of these research projects have been nominated and awarded the prestigious Franz Edelman Prize by the Institute of Operations Research and Management Science (INFORMS) for the practice of OR techniques. Winners of the Edelman award include American Airlines along with the predecessor to Sabre Airline Solutions for their work on Revenue Management and Continental Airlines and Caleb Technologies for their work on crew recovery.

The impact of operations research on the airline planning process has been and continues to be profound. Over the last 40 years operations research has played an integral role in many of the technological advancements credited to the airline industry. For instance, revenue management was developed using concepts from statistics, forecasting, and linear optimization. The various stages of the airline scheduling process have been modeled and implemented using ideas from network flow theory and mathematical programming. As researchers continue to make advances in these underlying fields of operation research, practitioners will be given additional tools to tackle unanswered business problems that exist in the industry. The continued improvement in commercial optimization software packages has encouraged and fostered the development of many of the state-of-the-art decision support tools now in use or currently under development. Such improvements, coupled with advances in computer hardware, will continue to push the horizon for OR practitioners in the airline industry.

Recent advances in global distribution system technology are impacting various airline business processes, especially revenue management. The availability of fare search engines that consider extensive flight networks to locate minimum fares will change the way pricing and revenue management analysts perceive and manage airline routes. Today, competition is no longer limited to obvious routings but an entire web of flight legs that are constantly changing as fares and availability change across networks. As a result, next-generation revenue management systems will have to be designed to cope with such network effects, which will increase the complexity of the underlying decision model to be solved. As airlines renew their existing fleets with crew compatible aircraft, the fleet assignment and crew-scheduling problems will increase in complexity, as the overall dimension of the feasible solution space increases. In situations where a carrier has various aircraft types with similar seating capacities, the potential for degeneracy while solving the fleet assignment problem increases, resulting in longer solution times.

Operations research has had a profound impact on the way airlines have evolved and competed in light of regulatory changes, from the current wave of deregulation and liberalization, to the introduction of global alliances. As we begin to consider hybrid scheduling problems and integrated airline scheduling, operation-research practitioners will further enhance and transform the way the airline industry functions. As these new scheduling concepts are embraced by airlines, they will continue to make change and improve their existing business processes and practices. Moving forward, the airline industry will be faced with future challenges that result from economic and geopolitical situations. The role of decision support systems based on OR techniques will be even more important for an airline's survival. The future mission of the airline operations-research community will be to deliver next-generation scheduling and planning tools that meet the prevailing needs of the industry and offer further opportunities for improved efficiencies, increased productivity, and enhanced creativity.

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